

# A dynamical origin for the top mass

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## ABSTRACT

We describe a dynamical scheme by which isospin breaking feeds strongly into the  $t$  and  $b$  masses and not into the  $W$  and  $Z$  masses. The third family feels a new gauge interaction broken close to a TeV.

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A large top quark mass presents an interesting puzzle to those who think that the origin of quark and lepton masses lies in gauge theory dynamics. How does the dynamics responsible for the isospin breaking in the  $t$ - $b$  masses not also feed into the  $W$  and  $Z$  masses and contribute substantially to  $\Delta\rho$ ? A typical approach to isospin breaking is to build it explicitly into the theory and have the right-handed  $t$  and  $b$  fields feel intrinsically different dynamics in the underlying theory. We will proceed in the opposite direction and contemplate a more dynamical origin of isospin breaking in the hope that this leads to more economical and appealing theories. In the technicolor context it was noticed that the dynamical techniquark masses can feel the effect of isospin breaking in physics at higher scales via effective 4-techniquark operators. This could lead to isospin breaking in techniquark condensates and thus quark and lepton masses. The trouble is that some fine tuning in the strength of the 4-techniquark operators is necessary to prevent this isospin breaking from contributing to  $\Delta\rho$ . This fine tuning becomes quite unbearable for a large top mass.[1]

In this note we will consider a mechanism which requires that the third family feel a new gauge interaction which is broken at relatively low scales, close to a TeV. In the picture we actually develop this breaking will coincide with electroweak symmetry breaking. We call this new interaction hypercolor (HC), and we will assume that it has a slowly walking gauge coupling.[2] The basic idea is that isospin breaking originates dynamically on energy scales of order 100 TeV and manifests itself in effective 4-hyperquark operators. One isospin breaking operator in particular remains relevant at lower energy scales due to the walking nature of HC. HC eventually breaks via a condensate which also breaks electroweak symmetry, and members of the third family become gauge singlets under the unbroken subgroup of HC. The 4-hyperquark operator, in combination with the condensate, then directly feeds a mass to the  $t$  and not the  $b$ . We will argue that because of the particular structure of the 4-hyperquark operator, neither it or other operators it induces will contribute significantly to isospin breaking in the electroweak breaking condensate.

The fermion fields participating in the electroweak breaking condensate will also be singlets under the unbroken subgroup of HC. In the end these fields are nothing more than an ordinary fourth family of fermions. The fourth family quarks,  $t'$  and  $b'$  will have masses of order 1 TeV and they will be approximately degenerate. Our goal

is to show how this can occur in a dynamical context while at the same time having large isospin breaking in the third family quark masses.

We will take  $SU(3)_H$  as the HC gauge group. There are two families of hyperfermions, each with hyperquarks and hyperleptons transforming under  $SU(3)_C \times SU(2)_L \times U(1)_Y$  in the standard way. One family of hyperfermions denoted by  $H$  all transform as a  $\mathbf{3}$  under HC while the other family,  $\underline{H}$ , all transform as  $\overline{\mathbf{3}}$ . The hyperquarks in these two families are  $Q = (U, D)$  and  $\underline{Q} = (\underline{U}, \underline{D})$ . The following condensates are assumed to occur on the TeV scale,

$$\langle \overline{\underline{U}}_L^A U_R^B \rangle \approx \langle \overline{\underline{D}}_L^A D_R^B \rangle \propto \delta^{A3} \delta^{B3} \quad (1)$$

where  $A$  and  $B$  are HC indices.<sup>2</sup> This breaks  $SU(2)_L \times U(1)_Y$  in the standard fashion. The condensate lies in the  $\mathbf{6}$  of  $SU(3)_H$  which thus breaks; we will call the unbroken subgroup  $SU(2)_M$  or metacolor. We will discuss the question of why this HC breaking condensate forms later. Whatever this dynamics is, it should preserve isospin to good approximation.

The HC triplets contain the fields  $(H^1, H^2, H^3) \equiv (M^1, M^2, f)$  and  $(\underline{H}^1, \underline{H}^2, \underline{H}^3) \equiv (\underline{M}^1, \underline{M}^2, \underline{f})$ .  $M^a$  and  $\underline{M}^a$  are two metacolored families and  $f$  and  $\underline{f}$  denote two families which are metacolor singlets. The latter contain the quark doublets  $q$  and  $\underline{q}$ . From the condensate we see that the Dirac spinors to be associated with the massive fourth family quarks are of the form  $[q_L, q_R]$ . From now on we denote these fields by  $q' = (t', b')$ . Dirac spinors of the form  $[q_L, \underline{q}_R]$  will be relabeled as the third family quarks  $q = (t, b)$ . Note that the two light families are HC singlets and are outside the present discussion.

Most of our discussion will also not involve the lepton sector, since the mechanism we are describing for the quarks need not also be occurring for the leptons. We will assume that the fourth family leptons are somewhat lighter than the fourth family quarks. Then the dynamical  $t'$  and  $b'$  masses make the main contribution to the  $W$  and  $Z$  masses and the associated decay constant is  $F \approx 145$  GeV. From this we may estimate the  $t'$  and  $b'$  masses,

$$m_{q'} \approx \sqrt{3} F \frac{m_\rho}{2f_\pi} \approx 1 \text{ TeV}. \quad (2)$$

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<sup>2</sup>Hermitian conjugate terms are implicitly assumed throughout this paper.

The  $\sqrt{3}$  accounts for  $m_{q'}$  scaling as  $N_c^{-1/2}$  when  $F$  is held fixed; in QCD  $N_c = 3$  whereas here  $N_c$  is effectively 1.

Now consider the presence of the isospin-violating (IV) but  $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$  preserving operator,

$$\frac{1}{\Lambda_{IV}^2} \epsilon_{ij} \bar{Q}_L^{Ai} D_R^A \bar{Q}_L^{Bj} U_R^B. \quad (3)$$

$SU(2)_L$  and HC indices are shown. This operator originates in “extended HC” (EHC) physics on energy scales of order 100 TeV. It cannot have a perturbative origin; it must arise dynamically as we discuss more below. This dynamics breaks  $SU(2)_R$  in a maximal way by not generating the related operator  $\epsilon_{ij} \bar{Q}_L^{Ai} U_R^A \bar{Q}_L^{Bj} D_R^B$ . Although operator (3) is generated at the EHC scale, we will take the actual value of  $\Lambda_{IV}$  to correspond to renormalization of the operator at the HC breaking scale.

The operator (3) contains the term

$$\frac{1}{\Lambda_{IV}^2} \epsilon_{ij} \bar{q}_L^i b'_R \bar{q}_L^j t_R. \quad (4)$$

In combination with the  $q'$  condensate (1) this produces a  $t$  mass, but no  $b$  mass. When the color indices appear in the appropriate way the result is

$$m_t \approx \frac{3 \langle \bar{b} b' \rangle}{4 \Lambda_{IV}^2}. \quad (5)$$

(There is no color sum inside the condensate.) The condensate is renormalized at the HC scale, and thus we estimate

$$\langle \bar{b} b' \rangle \approx 4\pi\kappa\sqrt{3}F^3. \quad (6)$$

$\kappa$  is a factor allowed to vary between 1 and 2, corresponding to a correction factor of 1.7 in QCD.[3] The  $\sqrt{3}$  accounts for the condensate scaling as  $N_c^{-1/2}$  when  $F$  is held fixed (see above and [4]).  $m_t = 175$  GeV would imply that

$$\Lambda_{IV} \approx (550 - 750) \text{ GeV}. \quad (7)$$

We see then that the coefficient of the 4-hyperfermion operator (3) is significantly smaller than a typical coefficient of a 4-hyperfermion operator produced by HC dynamics. By “naive dimensional analysis”, [5, 4] such a coefficient is of order  $1/F^2$ .

On the other hand this value for  $\Lambda_{IV}$  is consistent with this operator originating at the higher EHC scale if HC is a walking theory. HC interactions will generate an anomalous dimension of order  $2\gamma_m$  where  $\gamma_m$  is the anomalous dimension of either of the HC singlet mass operators  $\overline{Q}_L^A D_R^A$ ,  $\overline{Q}_L^A U_R^A$ . In a walking theory  $\gamma_m$  is close to unity.[2] Indeed at one loop in Landau gauge, operator (3) does not mix with other operators and its anomalous dimension is exactly equal to  $2\gamma_m$ . Thus we expect that operator (3) is strongly enhanced and it may be a relevant operator (or very nearly so) in the effective theory well below the EHC scale.

The crucial question is whether the presence of this operator implies that there are other significant contributions to  $\Delta\rho$  besides the one coming from the  $t$  mass. In fact operator (3) directly contributes to another  $SU(2)_R$  violating operator:

$$\frac{1}{\Lambda_{IV}'^2} \overline{Q}_L^A D_R^B \overline{D}_R^B Q_L^A. \quad (8)$$

This is generated from a one loop diagram with operator (3) and its hermitian conjugate at two vertices. If we repeat the procedure by putting operator (8) and its conjugate in a loop, then we can also generate the operator:

$$\frac{1}{\Lambda_{IV}''^2} [\overline{D}_R \gamma_\mu D_R]^2. \quad (9)$$

We again take the values of  $\Lambda_{IV}'$  and  $\Lambda_{IV}''$  to correspond to renormalization at the HC scale. Operator (8) directly contributes to the gap equation for the condensate (1), resulting in  $b'$ - $t'$  mass splitting. The latter is of order  $\langle \overline{b'}b' \rangle / \Lambda_{IV}'^2$ , and  $\Delta\rho$  is proportional to the square of this mass splitting in the standard way. Operator (9) directly induces the  $\Delta\rho$  term in the chiral Lagrangian,  $F^2 [\text{Tr}\{U^\dagger D_\mu U \tau_3\}]^2$ . This contribution to  $\Delta\rho$  is of order  $F^2 / \Lambda_{IV}''^2$ .

We have just said that the minimal contribution to the operators (8) and (9) implied by the existence of operator (3) occurs at the one and three loop level respectively. When the loop momenta in these diagrams is of order the EHC scale, the resulting effective operators must then be run down to the HC scale. But we see that the one-loop renormalization of these operators is completely different than operator (3). Besides causing mixing with other operators, the one-loop renormalization does not strongly enhance operators (8) and (9). This suggests that even beyond one loop,

the enhancement of operators (8) and (9) can be nonexistent or at least much less than the enhancement of operator (3). Considering also that the operators (8) and (9) may start off smaller at the EHC scale, it seems very plausible that  $\Lambda_{IV}'^2$  and  $\Lambda_{IV}''^2$  are sufficiently large to cause little contribution to  $\Delta\rho$ .

If we again consider the one loop diagram which generates the operator (8), we find also a long distance contribution when the loop momentum is of order the HC scale. Here naive dimensional analysis[5] can be used, which implies that the long distance contribution to  $1/\Lambda_{IV}'^2$  is of order  $F^2/\Lambda_{IV}^4$ . The resulting contribution to  $b'-t'$  mass splitting can then be written roughly as  $(F^2/\Lambda_{IV}^2)m_t$ . The important point is that it is small compared to the  $t$ - $b$  mass splitting, as is the corresponding contribution to  $\Delta\rho$ . Similarly, naive dimensional analysis implies that the long distance contribution to  $1/\Lambda_{IV}''^2$  is of order  $F^6/\Lambda_{IV}^8$  and is thus negligible.

The conclusion is that it appears natural for the largest contribution to  $\Delta\rho$  to be the one associated with the top quark mass.

In order to have some idea where the EHC operators come from we indicate a possible gauge group and particle content above the EHC scale.<sup>3</sup>[6, 7]

$$U(1)_A \times U(4)_S \times SU(4)_{PS} \times SU(2)_L \times SU(2)_R \quad (10)$$

$$\begin{aligned} & (+1, \mathbf{4}, \mathbf{4}, \mathbf{2}, \mathbf{1})_L \\ & (-1, \mathbf{4}, \mathbf{4}, \mathbf{1}, \mathbf{2})_R \\ & (-1, \overline{\mathbf{4}}, \mathbf{4}, \mathbf{2}, \mathbf{1})_L \\ & (+1, \overline{\mathbf{4}}, \mathbf{4}, \mathbf{1}, \mathbf{2})_R \end{aligned} \quad (11)$$

The “sideways”  $U(4)_S$  includes a  $U(1)$  with opposite vector charges for fermions in the  $\mathbf{4}$  and  $\overline{\mathbf{4}}$ . The important point is that we have a chiral gauge theory; any bilinear condensate breaks some gauge symmetry, even in the absence of the weak  $SU(2)_L \times SU(2)_R$ . Because this is not a vector-like theory we avoid the Vafa-Witten result,[8] and can thus contemplate breaking isospin via the dynamical breaking of  $SU(2)_R$ . We take one manifestation of this to be EHC scale Majorana masses for the right-handed neutrinos and hyperneutrinos, which leaves unbroken the correct hypercharge  $U(1)_Y$ . These condensates will be the only bilinear ones allowed by the unbroken symmetries.

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<sup>3</sup>The Pati-Salam gauge group, “422”, may actually be replaced by some subgroup which contains “321”.

We take another manifestation to be the  $SU(2)_R$  violating condensate

$$\langle \epsilon_{ij} \overline{Q}_L^{Ai} D_R^A \overline{Q}_L^{Bj} \underline{U}_R^B \rangle. \quad (12)$$

This signals the dynamical generation of a 4-point function which otherwise vanishes in perturbation theory. For external momenta small compared to the EHC scale, the momentum dependence of the 4-point function should be well described by the effective theory in which the corresponding local operator times a coefficient of order  $1/\Lambda_{EHC}^2$  appears. The result at HC scales is the desired effective operator (3).

We argue that the appearance of condensate (12) can be considered natural (in the sense of no fine-tuning), and we may illustrate this point by a toy scalar field potential. Consider a scalar field  $\phi_{ik\underline{i}\underline{k}}$  where  $i$  and  $\underline{i}$  are  $SU(2)_L$  doublet indices and  $k$  and  $\underline{k}$  are  $SU(2)_R$  doublet indices. We then construct the most general  $SU(2)_L \times SU(2)_R$  invariant, quadratic plus quartic scalar potential. An underlined index cannot be contracted with an index not underlined; this reflects the two flavors of hyperquark doublets in the model. For a range of values for the parameters we find that at the minimum of the potential  $SU(2)_R$  breaks but not  $SU(2)_L$ . The vacuum expectation value could be<sup>4</sup>

$$\langle \phi_{ik\underline{i}\underline{k}} \rangle \propto \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}_{\underline{i}\underline{i}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}_{k\underline{k}}, \quad (13)$$

which corresponds to our desired condensate (12).

Below the EHC scale we take the unbroken gauge symmetry to be<sup>5</sup>

$$U(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (14)$$

Operators of interest for the 1 TeV HC breaking and which can be produced perturbatively at the EHC scale are

$$\overline{Q}_L^A Q_R^A \overline{Q}_R^B Q_L^B \quad \text{and} \quad \overline{\underline{Q}}_L^A Q_R^A \overline{\underline{Q}}_R^B \underline{Q}_L^B. \quad (15)$$

Like operator (3) they are strongly enhanced as they are run down to the HC scale, and can thus play a role in the HC symmetry breaking. If the contribution from the

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<sup>4</sup>Weak vacuum alignment may help choose the direction of the vev.

<sup>5</sup>The  $U(1)_A$  must be broken since it couples to light fermions; this can be accomplished by the EHC condensate  $\langle \overline{Q}_L^A Q_R^A \overline{\underline{Q}}_R^B \underline{Q}_L^B \rangle$ .

massive  $U(1)_A$  boson dominates then these operators are produced with the desired signs to resist the formation of the HC singlet condensates  $\langle \overline{Q}_L^A Q_R^A \rangle$  and  $\langle \overline{Q}_L^A \underline{Q}_R^A \rangle$ . It is this  $U(1)_A$  effect which can drive the HC dynamics away from what would naively be the most attractive channel; we assume that the HC dynamics instead produces the HC breaking condensate (1). This latter condensate leaves unbroken the gauge symmetry

$$U(2)_M \times SU(3)_C \times U(1)_{EM}. \quad (16)$$

We have been describing possible dynamics responsible for the mass of a fourth family and the top quark. Is it at least conceivable that all the other quark and lepton masses can be generated within this same scheme? One possibility has been described in [7]. We would like to briefly describe an alternative way to generate light fermion masses involving slightly less breaking of the original gauge symmetries. The result is the massless metaphoton in  $U(2)_M$  which would have interesting implications of its own.[9] We relegate this discussion to an Appendix since it is quite independent of the main point of this paper.

The main point has been to develop a dynamical scheme by which a large top mass is consistent with a small  $\Delta\rho$ . The scheme appears to be both natural and economical. Of course it is impossible to claim that the strongly interacting chiral gauge theory actually behaves the way we have described. But the scheme does present a testable picture of the physics in the 0.1 to 1 TeV range. Besides the fourth family there is the metacolor sector,[7] which produces a mass spectrum of bound states lying in this energy range. There is also a massive hypercolor gauge boson (the diagonal generator in  $U(3)_H/U(2)_M$ ) which couples only to the third and fourth families.



## Appendix

As described in [7] we will assume that no dynamical metafermion mass forms, which is guaranteed if certain discrete symmetries remain unbroken. In this situation we expect little contribution to the electroweak correction parameter  $S$  from the metacolor sector. 4-metafermion condensates are sufficient to produce the  $b$  and  $\tau$  masses, and the  $e$ ,  $\mu$ , and  $\tau$  neutrinos are naturally light. The new feature we are considering is the metaphoton which has opposite vector charges for the two metacolored families. A massless metaphoton did not exist in [7] since in that case the generation of masses for the light two families involved operators generated at the EHC scale which broke the original  $U(1)$  in  $U(4)_S$ . We must therefore find another way to generate masses for the two light families.

We will denote members of the two light families by  $\chi$  and  $\underline{\chi}$  (which lie in the  $\mathbf{4}$  and  $\overline{\mathbf{4}}$  in (11)). We first note that their masses depend on HC breaking effects. To the extent that EHC interactions are HC conserving then operators of the form  $\overline{H}_L H_R \overline{\chi}_R \chi_L$  and  $\overline{\underline{H}}_L \underline{H}_R \overline{\underline{\chi}}_R \underline{\chi}_L$  are generated by broken  $SU(4)_S$  gauge boson exchange. But these are useless for feeding down mass from the third or fourth family quarks to the first two families since the  $t'$  and  $b'$  masses are of the form  $\overline{\underline{H}}_L^3 H_R^3$  while the  $t$  and  $b$  masses are of the form  $\overline{H}_L^3 \underline{H}_R^3$ .

But the question is whether EHC physics is completely immune to the HC breaking physics. The walking nature of the HC interaction suggests that perhaps it is not. We will entertain the notion that HC breaking should be reflected in the effective theory all the way up to EHC scales, with the only constraint being that the HC breaking effects be consistent with HC gauge boson masses less than a TeV.

We consider the existence on EHC scales of the HC violating operators

$$\overline{H}_L^3 \underline{H}_R^3 \overline{\underline{\chi}}_R \chi_L \quad \text{and} \quad \overline{\underline{H}}_L^3 H_R^3 \overline{\chi}_R \underline{\chi}_L. \quad (17)$$

These operators will be produced for example if there is a mass splitting between a certain massive  $SU(4)_S$  gauge boson in the  $SO(4)$  of  $SU(4)_S$  and an associated gauge boson not in the  $SO(4)$ . In any case the size of the coefficients of the operators is of order  $1/\Lambda_{EHC}^2$  times some suppression factor. The suppression factor, perhaps of order 0.1 or 0.01, ensures that the feedback into the massive HC gauge bosons is sufficiently small.

These operators will feed mass down to the two light families. Note that the Dirac spinors of one light family have the form  $[\chi_L, \underline{\chi}_R]$  and the other light family,  $[\underline{\chi}_L, \chi_R]$ . As of yet there is no mass mixing to produce weak mixing angles. (Nor are there flavor changing neutral currents.) Mass mixing between the two heavy families and the two light families can be induced by other HC violating operators, for example  $\overline{H}_L^3 \underline{H}_R^3 \overline{H}_R^3 \chi_L$ . Such effects cannot be large, which is consistent with the observed smallness of the KM mixing involving the third family. Finally we note that the origin of Cabibbo mixing between the first two families may be associated with the large right-handed neutrino masses,  $\nu_{eR}\nu_{eR}$ ,  $\nu_{\mu R}\nu_{\mu R}$ ,  $\nu_{eR}\nu_{\mu R}$ . For example these masses could communicate with the quark sector via  $SU(4)_{PS}$  interactions, thus producing  $u$ - $c$  mass mixing.  $d$ - $s$  mass mixing and thus  $K^o\overline{K}^o$  mixing is naturally suppressed.[10]

We are feeding down masses to the light fermions via operators generated at the EHC scale. Each light mass will thus be determined by the mass of the appropriate fermion of the third or fourth family evaluated at the EHC scale. Notice that the various third and fourth family fermions are receiving masses in quite different ways, thus leading to quite different masses at the EHC scale. For example, it is interesting to speculate that the fourth family masses are actually much softer at high energies than the third family masses. In this case the first and second families could receive their masses from the fourth and third families, respectively. This would produce more isospin breaking in the second family than in the first, as observed.

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